

PATTERNS OF CONSTRAINED PARTICLE SETTLING IN WATER MINERAL SUSPENSIONS OF DIFFERENT DENSITIES

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Abstract. The settling velocity of particles in mineral suspensions is a crucial parameter for calculating the design of various hydraulic devices and equipment used for mineral pulp beneficiation. In studies of gravity separation of heterogeneous particles by settling, the determination of mass settling velocity, the influence of suspension density on the process, and the applicability of classical hydrodynamics laws remain the least explored aspects. Often, free settling conditions are used for calculating hydraulic separation processes, but this introduces significant error in the velocity magnitude, as, in practice, the process occurs under constrained conditions. The purpose of this work was to analyze the patterns of constrained settling using the example of coal particle settling in fly ash suspensions from thermal power plants. The article employs an original method for calculating the characteristics of suspensions and the velocity of constrained settling depending on the density. Experimental data on the mass settling velocity of natural fly ash are presented, which indicate the order of velocities and give grounds for the velocity calculation. Given the fine particle size of the ash, the main focus was on the settling of fine coal in the ash. The analysis covered a database in which the characteristics of suspensions and the velocity of constrained coal settling were determined by varying the density of the ash suspension from 1.05 g/cm³ to 1.3 g/cm³ and the size of the settling coal from 0.01 mm to 4 mm. The database was analyzed using the Reynolds number and the applicability of Stokes' law and Lyashenko's law. It was found that the more dilute is the suspension, the smaller is the particle size that follows Stokes' law, and the smaller is the range of particle sizes that Stokes' law covers, and vice versa. For fine coal fractions of 0.001–0.1 mm, the numerical coefficient in Stokes' law decreases according to an inverse power law depending on the pulp density. The ratio of free to constrained settling velocities decreases according to a power law, similar to Lyashenko's law for porosity. The conducted research expands scientific understanding of the processes of constrained settling, facilitates engineering calculations when designing hydraulic devices, and optimizes their operational modes.

Keywords: suspension, fly-ash, density, speed, constrained deposition.

1. Introduction

The processing and utilization of industrial anthropogenic waste is a pressing issue today [1]. In this regard, significant attention is drawn to the multi-million-ton deposits of waste from thermal power plants (TPPs), which consist of 80–90% fly ash. The primary focus is on the use of ash as a building material in the production of concrete, bricks, and other materials [2, 3]. Currently, only 10–12% of ash is used in this way in Ukraine. The main obstacle is the high content of unburned coal in the ash, up to 20–25%, whereas construction standards require it to be reduced to 5–10% [4].

At the IGTM of the NAS of Ukraine, methods and means for the hydraulic removal of coal from fly ash are being developed by creating an upward flow in hydraulic devices [5]. Hydraulic methods, unlike flotation, are more cost-effective, productive, and environmentally safe. Due to the upward flow velocity, lighter and finer particles are carried into the overflow of the hydroseparator, while heavier and coarser particles settle into the sands. The relatively large working surface area of the devices ensures high productivity. It is evident that the upward flow velocity should match the settling velocity of the particle type and size that needs to be carried into the overflow.

The determination of the constrained settling velocity of heterogeneous particles in mineral suspensions (pulp) is an important theoretical and practical task. Based on



this velocity, beneficiation indicators of mineral pulps are predicted, the design of various hydraulic separators is calculated.

A distinctive feature of the hydraulic separators is that for the same particle, the settling behavior can shift from laminar to laminar-turbulent depending on the density of the medium, making the commonly known free settling formulas [6] often inapplicable. The process of constrained sedimentation has been studied very little [6]. This article examines the laws of constrained sedimentation using coal and quartz particles in ash suspension for the implementation of a method of removing coal from ash due to the upward flow.

The analysis of coal and quartz particle movement in ash was first investigated in [7], but the density of the ash solid phase was assumed to be 2.98 g/cm^3 , which does not correspond to the actual experimentally measured value of 2.0 g/cm^3 [8]. Additionally, the fundamental constrained settling patterns from the perspective of classical hydrodynamics, such as the features of Reynolds number, applicability of Stokes' and Lyashenko's laws, remained undefined. The purpose of this work is to establish the general patterns of constrained settling using the example of coal particle sedimentation in fly ash suspensions from TPPs, which is required for design and research work, development of methods and apparatus for hydroseparation of finely dispersed mineral pulps.

2. Methods

The study employed original experimental and theoretical methods to determine the characteristics of mineral suspensions and hydraulic settling processes [8–10], as well as classical definitions and laws of hydrodynamics [6].

The research methodology used in the article pertains to the settling of coal in fly ash suspensions from TPPs, but it is also suitable for analyzing settling in any mineral suspension.

The methodology involves the sequential solution of the following tasks:

1. Experimental determination of the solid phase density of the composite fine-grained mixture that constitutes the aqueous mineral suspension [8].
2. Determination of the hydraulic characteristics of suspensions with specific density values [9].
3. For each density value, calculation of the constrained settling velocity of particles of a specific type at various sizes [10].

Based on this, a database is created to analyze the settling process, in which the constrained settling velocity is determined for specific suspension densities and settling particles at several given density values by varying the particle size.

The advantage over other research methods is that this methodology requires only two experimental measurements: the solid phase density of the suspension and its density or bulk weight, which can be easily measured by weighing a sample of the suspension in a measuring container.

In the article, constrained settling is analyzed using the example of coal particles with a density of 1.5 g/cm^3 , varying in size from 0.01 mm to 4 mm in a fly ash suspension with a solid phase density of 2 g/cm^3 and a varied density of 1.05 g/cm^3 to

1.3 g/cm³. The target function is the velocity. The obtained database was analyzed using correlation analysis methods and the freely available Mc. Excel software package.

3. Results and discussion

Key definitions and formulas. For an arbitrary composite aqueous suspension containing fine particles with different properties, the solid phase density ρ_m is one of the most important characteristics. In the study [7], it was shown that the theoretical determination of ρ_m is not adequate to the true density of the mixture. This indicator can only be reliably determined experimentally by filling the gaps between the particles with water. For fly ash from TPPs, authors established that ρ_m can be taken as 2 g/cm³ [8].

To calculate the constrained settling velocity for a given suspension density ρ_s it is necessary to know its hydraulic characteristics [9]. The concentration of the suspension β is expressed through the volume fraction of liquid in the suspension ε , which is called porosity ε . The kinematic viscosity ν (when $\nu_0=0.01$ cm²/c) and the exponent Δ are also required. These indicators are determined by the formulas (1), [9]:

$$\varepsilon = \frac{\rho_m - \rho_s}{\rho_m - 1}, \quad \beta = 1 - \varepsilon, \quad \nu = \nu_0 \cdot \exp \frac{2.5 \cdot \beta + 0.675 \cdot \beta^2}{1 - 0.609 \cdot \beta}, \quad \Delta = \frac{\rho_p - \rho_s}{\rho_s}. \quad (1)$$

The characteristics of the medium are the same for the movement of different settling particles, except for the parameter Δ , where the specific density of a particular particle type is involved. For coal, this is $\rho_p=1.5$ g/cm³. The characteristics of ash suspensions with a solid phase density $\rho_m=2.003$ g/cm³ calculated according to formulas (1) are presented in Table 1

Table 1 – Hydraulic characteristics of fly ash suspension, $\rho_m=2.003$ g/cm³

Density, ρ_c , g/cm ³	Porosity ε , units	Concentration, β , units	Viscosity, ν , cm ² /s	Δ_c , units
1.05	0.95	0.05	0.01140	0.429
1.1	0.90	0.10	0.01314	0.364
1.15	0.85	0.15	0.01536	0.304
1.2	0.80	0.20	0.01822	0.250
1.25	0.75	0.25	0.02197	0.200
1.3	0.70	0.30	0.02697	0.154

To calculate the constrained settling velocity, we use the C. Ergun equation, the justification for which is provided in [10]. Taking into account that $g = 981$ cm/s², we can write it in a simplified form for easier calculations:

$$\omega^2 \cdot d + 85.71 \cdot \omega \cdot (1 - \varepsilon) \cdot \nu - 560.57 \cdot d^2 \cdot \varepsilon^3 \cdot \Delta = 0, \quad (2)$$

where ω is given in cm/s, d' – in cm.

Equation (2) in this study is solved both for ω , and for d , when ω is measured, the parameters ε , v , Δ are determined by formulas (1) and presented in Table 1.

Experimental part. To validate equation (2), experiments were conducted to measure the mass settling velocity of ash. The results of the mass settling velocity measurements are presented in Table 2 and in Figure 1.

Table 2 – Experimental determination of ash suspension characteristics and mass settling velocity of natural ash, $\rho_m = 2.003 \text{ g/cm}^3$

$\rho_s \text{ exper., g/cm}^3$	$\varepsilon \text{ exper., units}$	$\omega \text{ exper., mm/s}$
1.31	0.69	0.15
1.23	0.75	0.22
1.21	0.79	0.26
1.15	0.83	0.35
1.11	0.87	0.53

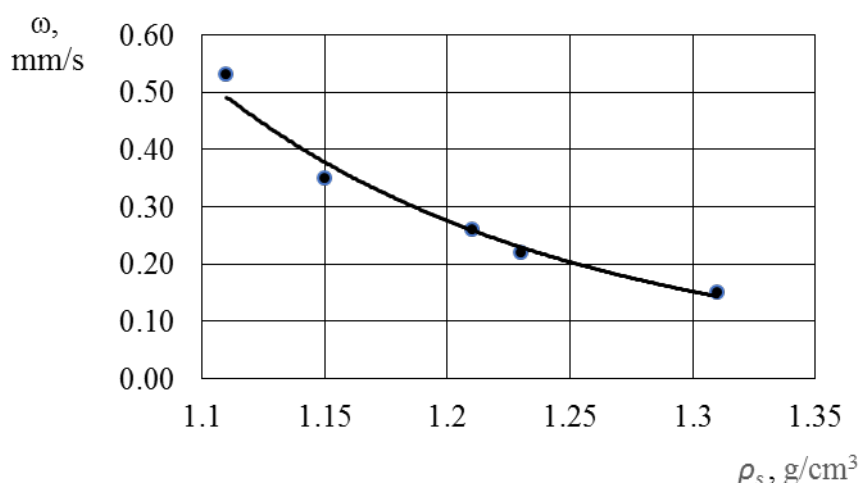


Figure 1 – Experimental dependence of the mass settling velocity of natural ash on the density of the ash suspension

The data in Table 2 indicate a rather slow settling of ash even in dilute suspensions. The dependence in Figure 1 is approximated by a shallow inverse power function:

$$\omega_{\text{exper.}} = 1,0654 \cdot \rho_s^{-7,414}, R^2 = 0.976.$$

Each density and velocity correspond to a characteristic size of ash, coal, and quartz particles that settle all together. Therefore, using the velocity and data from Table 2, the corresponding sizes of heterogeneous particles in the suspension were calculated using formula (2). Here v was calculated using formulas (1), and the parti-

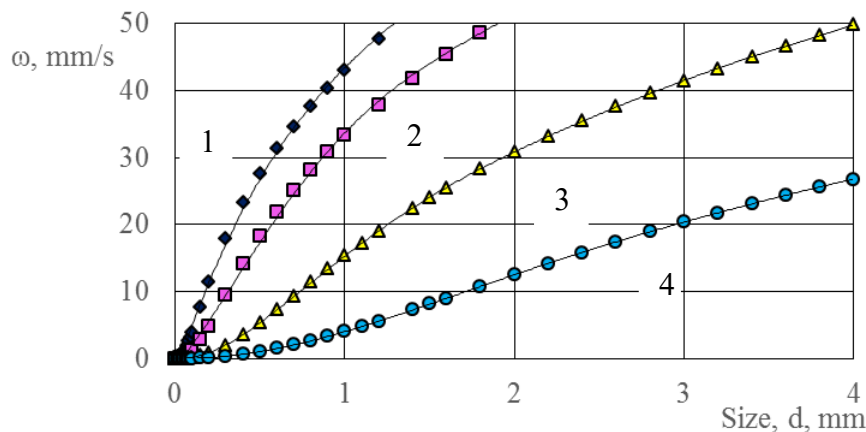
cle density was taken as follows: for ash $\rho_p=2.003 \text{ g/cm}^3$, coal $\rho_p=1.5 \text{ g/cm}^3$, and quartz $\rho_p=2.65 \text{ g/cm}^3$ (Table 3).

Table 3 – Characteristic sizes of heterogeneous particles during mass settling of ash

ρ_s exper., g/cm ³	ε exper., units	v , cm ² /s	ω exper., mm/s	Ash		Coal		Quartz	
				Δash , units	d_{ash} , mm/s	Δc , units	d_c , mm/s	Δq , units	d_q , mm/s
1.31	0.69	0.02816	0.15	0.53	0.107	0.15	0.204	1.02	0.077
1.23	0.75	0.02197	0.22	0.63	0.083	0.22	0.125	1.15	0.054
1.21	0.79	0.01889	0.26	0.66	0.070	0.24	0.115	1.19	0.052
1.15	0.83	0.01642	0.35	0.74	0.059	0.30	0.081	1.30	0.039
1.11	0.87	0.01441	0.53	0.82	0.053	0.36	0.063	1.41	0.032
Average					0.075	-	0.118	-	0.051

The data in Table 3 show the difference in the characteristic sizes of ash itself and the particles within it, even though they all settle at the same speed. Under the same conditions, the quartz particles being settled are the largest, on average 2.3 times larger than the coal particles. Important is that all sizes shown in Table 3 correspond to the actual sizes according to the granulometric analysis of ash, which unequivocally confirms the validity of Ergun's equation (2).

Analytical part. To remove coal from ash using hydraulic methods, it is necessary to analyze the settling patterns of coal in ash. For this purpose, a database was prepared using Table 1 and equation (2), with the following parameters varied: coal size is 0.001–4 mm with arbitrary steps, and density is 1.05, 1.1, 1.2, 1.3 g/cm³. Higher densities are not analyzed because at 1.35 g/cm³, the aqueous ash mixture becomes a moist mass and loses its suspension properties. The dependencies $\omega = f(d)$ for coal movement in ash are shown in Figure 2.



1 – 1.05; 2 – 1.1; 3 – 1.2; 4 – 1.3 (g/cm³)

Figure 2 – Constricted settling velocity of coal depending on size in ash suspensions of different densities, ρ_s

The dependencies in Figure 2 have low approximation accuracy even with sixth-degree polynomials, so each was divided into two parts—separately for fine fractions (Figure 3, a) and separately for coarse fractions (Figure 3, b).

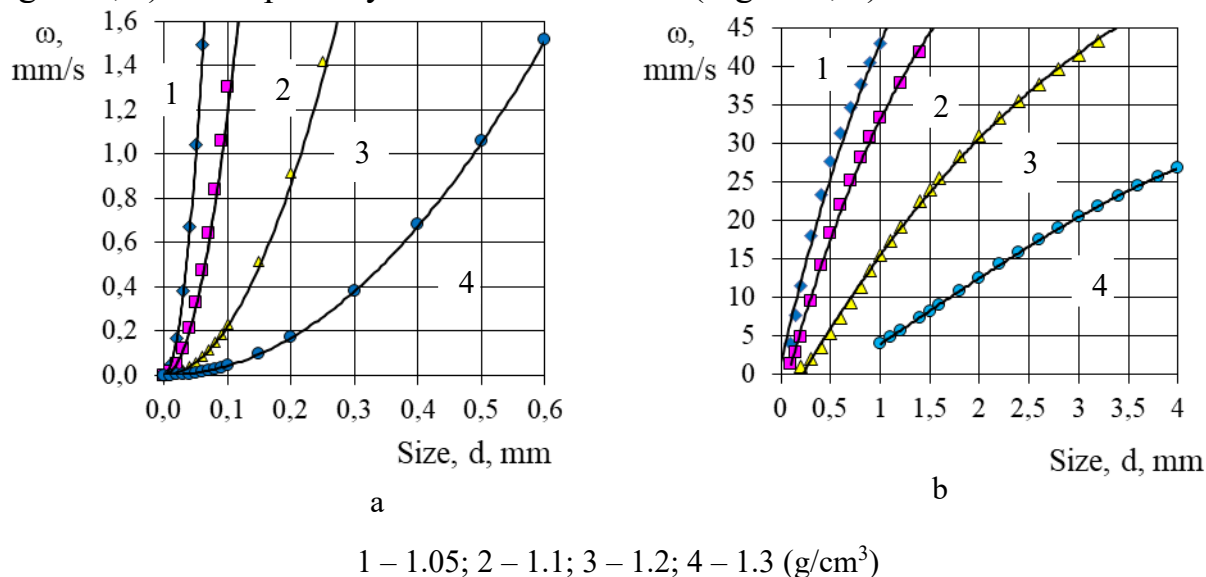


Figure 3 – Settling velocity of fine (a) and coarse (b) coal ρ_s

The dependencies for fine coal in Figure 3a are well approximated by a quadratic parabola, meaning that the settling behavior follows Stokes' law:

$$\omega = \frac{K \cdot d^2}{\nu}.$$

Let $k = K/\nu$, so in further calculations, we will use the numerical coefficient for d^2 , where k has the dimension of 1/mm·s.

Based on the obtained database, the value of k for each specific density was first determined using an approximating quadratic correlation equation, and then refined manually to ensure the best match with calculations using formula (2). The particle size range where Stokes' law applies was established, and the calculated velocity formulas were obtained (Table 4).

Table 4 – Stokesian constrained settling of fine coal (Figure 3a) in ash suspension

Pulp Density ρ_s , g/cm ³	Coal Size d , mm	Velocity Formula ω , mm/s	Viscosity ν , mm ² /s	For max size d , mm	
				ω , mm/s	Re= $\omega \cdot d/\nu$, units
1.05	$d \leq 0.1$	$391 \cdot d^2$	1.140	3.91	0.343
1.1	$d \leq 0.15$	$131 \cdot d^2$	1.314	2.95	0.336
1.15	$d \leq 0.2$	$52.8 \cdot d^2$	1.536	2.11	0.275
1.2	$d \leq 0.3$	$22.8 \cdot d^2$	1.822	2.05	0.338
1.25	$d \leq 0.5$	$10 \cdot d^2$	2.197	2.5	0.569
1.3	$d \leq 1.0$	$4.2 \cdot d^2$	2.697	4.2	1.557

1.4	$d \leq 1.5$	$0.57 \cdot d^2$	4.327	1.28	0.445
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Table 4 presents simple calculation formulas for Stokesian constrained settling of coal for specific values of ash suspension density. When solving certain problems, it may be necessary to determine the particle size for a given upward flow velocity in the working zone of a hydraulic device. These dependencies for fine coal are determined from Table 4 as $d = f(\omega)^{0.5}$. At the same time, a restriction on velocity is established according to the maximum particle size d , corresponding to a given density.

The velocity formulas from Table 4 are also suitable for the movement of hollow microspheres—fused quartz particles with a density of 2.65 g/cm^3 , in which part of the volume is occupied by air. The density of such a microsphere, like that of coal, is 1.5 g/cm^3 , and the percentage of air is determined as x in the equation: $2.65 \cdot (100-x) = 1.5 \cdot 100$ and is equal to 43.4%.

Analysis of constrained particle settling from Reynolds number. Table 4 shows the velocity and Reynolds number Re for the largest particles that fall within the Stokesian range from column 2. These are the maximum limiting values of ω and Re . It is known [6] that laminar motion according to Stokes' law occurs when the Reynolds number $Re < 0.5$. However, Table 4 shows that the condition $Re < 0.5$ can be violated, but the Stokesian settling law still holds.

It was established that for particles of a certain size, the dependence of Re on pulp density follows a power law (Figure 4).

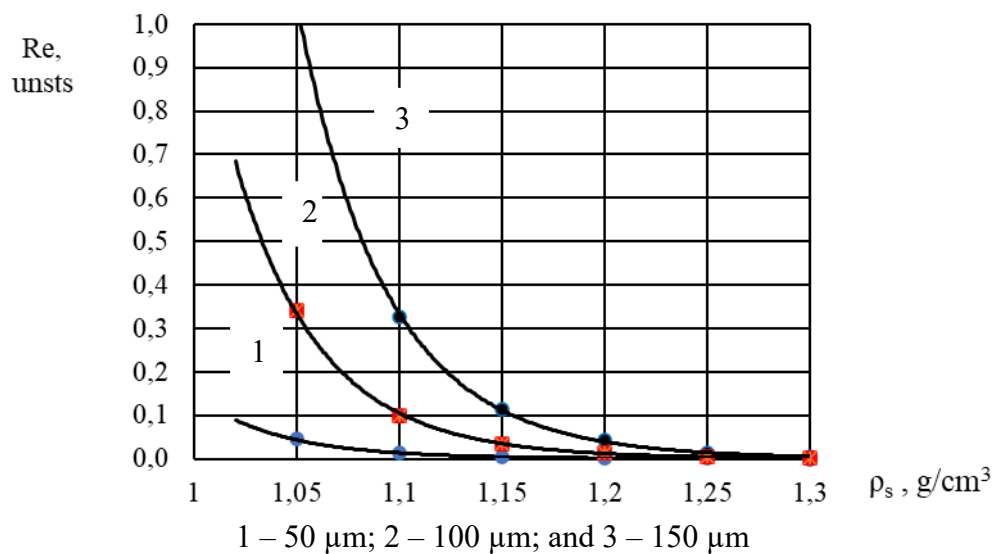


Figure 4 – Reynolds number Re for different pulp densities during coal settling in ash with particle sizes

The control value $Re = 0.5$, according to dependencies 1–3, is observed for a particle size of 50 μm at any density, and for 100 μm at a density of 1.03 g/cm^3 and higher, and for 150 μm at 1.08 g/cm^3 and higher. Under these conditions, $Re < 0.5$, the motion is laminar, and it follows Stokes' law.

For coarse coal particles of 100–150 μm at $\rho_s \rightarrow 1$ (water density), $\text{Re} > 0.5$, and the settling takes on a turbulent character.

Thus, for coarse coal particles, the Stokesian nature of motion is observed only under conditions of increased ash suspension density.

It should be noted that settling in pure water is usually described by Stokes' law, but this is not always accurate. For example, the free settling velocity of 1 mm quartz particles in water, where $\nu = 0.01 \text{ cm}^2/\text{s}$, is 9.5–10 cm/s [6, 10]. In this case, $\text{Re} = 95\text{--}100$ units, meaning the settling does not meet the condition $\text{Re} < 0.5$, it is not laminar, and cannot be described by Stokes' law.

The fact that Stokesian settling of coal is observed under conditions of slightly increased pulp density is explained by the mixed nature of the movement in constrained conditions, which is both laminar and turbulent. Coarse coal can settle either with flow separation from the particle or without it. Flow separation is characteristic for turbulent movement and occurs in dilute suspensions for coarse particles. When the density is slightly increased, settling can occur without flow separation, which is characteristic for laminar movement.

Thus, the Reynolds number, as a criterion for the application of Stokes' law, is conditional. As shown in Table 4, the condition $\text{Re} < 0.5$ can be violated, but the particle's motion still follows Stokesian behavior.

Limitations on the application of Stokes' law. Table 4 shows that for each density, the applicability of Stokes' law is limited by the particle size of coal. The higher is the pulp density, the larger is the particle size for which Stokes' law applies. For example, in dense suspensions of 1.3 g/cm³ and 1.4 g/cm³, Stokes' law is valid for coal particles ranging in size from 0.001 mm (10 μm) to 1 mm and 1.5 mm.

It is also important to note that Stokes' formula has limitations not only on the upper particle size but also on the lower size. It is not applicable when Brownian motion dominates over settling motion, which occurs with coal particles of 5–10 μm in size. As the density increases, the numerical coefficient for d^2 , as well as the velocity, significantly decreases.

The following pattern was established: *the more is diluted the suspension, the smaller is the particle size for which Stokes' law applies, and the narrower is the size range covered by Stokes' law, and vice versa.*

Analysis of the numerical coefficient in Stokes' law. Fly ash pulp consists of particles with sizes up to 250 μm . For particles of 100 μm and smaller, a general law of the numerical coefficient's decrease in Stokes' law with density can be established.

It was found that for fine coal fractions of 0.001–0.1 mm, with increasing medium density, the numerical coefficient in Stokes' law decreases according to an inverse power law (Figure 5):

$$k_{\text{coal}} = 1188.6 \cdot \rho_s^{-22.09}, \quad R^2 = 0.9993.$$

A similar dependence was obtained for the settling of quartz particles with a size of $d \leq 0.1 \text{ mm}$ depending on the density of the ash suspension:

$$k_{quartz} = 2918,5 \cdot \rho_s^{-17,86}, \quad R^2 = 0.993.$$

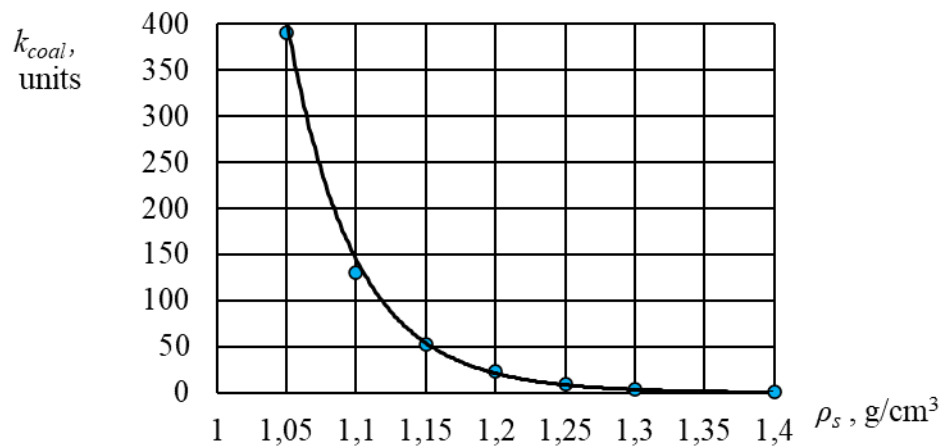


Figure 5 – Dependence of the numerical coefficient k in Stokes' law $\omega_{coal} = k \cdot d^2$ on the suspension density for the movement of coal particles with size $d \leq 0,1 \text{ mm}$

Thus, the reduction of the numerical coefficient in Stokes' law according to the inverse power law is of a general nature.

Comparison of free and constricted settling. It is known that the free settling velocity according to Stokes is determined by the formula [6]:

$$\omega = \frac{g}{18} \cdot \frac{d^2}{\nu} \cdot \frac{\rho_p - \rho_s}{\rho_s} = 54.481 \cdot \frac{d^2 \cdot \Delta_c}{\nu}, \quad (3)$$

where ω – cm/s, d – cm, $g = 980.66 \text{ cm/s}^2$, ν and Δ_c are given in Table 1.

Using formula (3) and the data from Table 1, the numerical coefficient for d^2 and the ratio of free to constricted settling velocities were calculated (Table 5).

Table 5 – Comparison of free and constrained settling of coal in ash suspensions, $\rho_m = 2 \text{ g/cm}^3$

Pulp Density ρ_c , g/cm ³	Settling Velocity ω , mm/s; d, mm		Ratio of Free to Constricted Settling Velocities (Table 2)
	Free	Constrained	
1.05	$204.89 \cdot d^2$	$391 \cdot d^2, d \leq 0.1$	0.52
1.1	$150.72 \cdot d^2$	$131 \cdot d^2, d \leq 0.15$	1.15
1.15	$107.925 \cdot d^2$	$52.8 \cdot d^2, d \leq 0.2$	2.04
1.2	$74.74 \cdot d^2$	$22.8 \cdot d^2, d \leq 0.3$	3.28
1.25	$49.6 \cdot d^2$	$10 \cdot d^2, d \leq 0.5$	4.96
1.3	$31.08 \cdot d^2$	$4.2 \cdot d^2, d \leq 1.0$	7.40
1.4	$8.994 \cdot d^2$	$0.57 \cdot d^2, d \leq 1.5$	15.78

With increasing density, the difference between free and constrained settling increases. For a typical density of hydraulic devices of $1.1 \div 1.2 \text{ g/cm}^3$, the free settling

rate according to formula (3) is higher than constrained by $1.15 \div 3.28$, on average by 2.2 times. The dependence of the velocity ratio on density is shown in Figure 6.

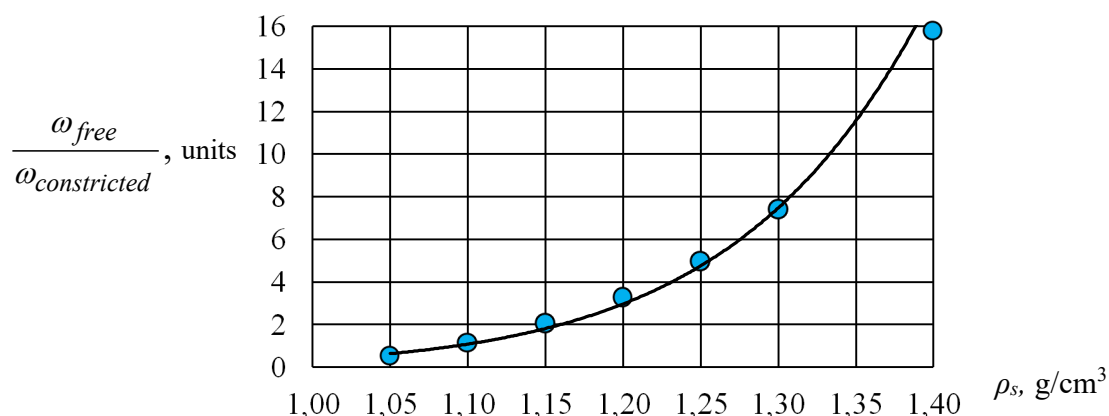


Figure 6 – Ratio of free to constricted settling velocities of coal for different fly ash suspension densities

Regularity was established that for fine coal fractions with $d \leq 0.1$ mm, *the ratio of free to constricted settling velocities increases according to a power law as the pulp density increases:*

$$\Omega_{coal} = \frac{\omega_{free}}{\omega_{constricted}} = 0.3622 \cdot \rho_s^{11.542}, \quad R^2 = 0.9964. \quad (4)$$

A similar pattern was obtained for the precipitation of quartz with a particle size of $d \leq 0.2$ mm, $\rho_p = 2.65$ g/cm:

$$\Omega_{quartz} = \frac{\omega_{free}}{\omega_{constricted}} = 0.3732 \cdot \rho_c^{11.414}, \quad R^2 = 0.9948.$$

Thus, the power law of the ratio of velocities of free to constricted settling from density is general.

Applicability of Lyashenko's law. According to the well-known law of P.V. Lyashenko, the ratio of free to constricted settling velocities is given by [6]:

$$\frac{\omega_{free}}{\omega_{constricted}} = \varepsilon^{-\lambda}, \quad (5)$$

where ε is the porosity of the suspension (volume fraction of gaps between particles), and the exponent λ is determined experimentally and depends on many factors.

Based on the obtained data on coal settling in ash suspensions, a correlation dependence was established:

$$\Omega_{coal} = \frac{\omega_{free}}{\omega_{constricted}} = \varepsilon^{-\lambda} = 9.014 \cdot \varepsilon^{-5.697}, \quad R^2 = 0.989. \quad (6)$$

Thus, the application of Lyashenko's law (5) requires the determination not only of λ , but also of a constant numerical coefficient, as shown in (6).

The equation (4) obtained above is analogous to Lyashenko's law (6). The difference is that the density ρ_s in equation (4) is easily measured by weighing a pulp sample in a liter container, whereas determining ε in equation (5) requires significantly more measurements [8, 9]. The approximation accuracy of equation (4) is slightly higher than that of (5), as indicated by the R^2 value. For both equations (4) and (6), the limitation on particle size shown in Table 4 must be met for a given density.

Thus, the analytical studies on constrained settling of coal in fly ash suspensions allowed us to establish the limits of applicability of the quadratic Stokes law, derive simple calculation formulas for this law for a given pulp density, establish the inverse power dependence of the numerical coefficient in Stokes' law, and the power law of the increase in the ratio of free to constrained settling velocities with increasing pulp density. The obtained data simplify the calculation of constrained settling velocities of coal in fly ash suspensions, which is necessary for designing and determining the operating modes of hydraulic separators for cleaning ash from harmful coal impurities to use it as a high-quality building material.

4. Conclusions

Gravity beneficiation methods for mineral processing are based on the different settling velocities of particles of varying sizes and densities in aqueous mineral suspensions. Correct determining the constrained settling velocity depending on pulp density is a relevant scientific and practical task.

This study employs an original methodology for investigating constrained settling across a wide range of mineral pulp densities with mixed laminar-turbulent flow behavior. It is for the first time when the following new results were obtained for constrained sedimentation.

It is shown that using the Reynolds number as a criterion for laminar flow is conditional. For a given ash density and coal particle size, the criterion $Re < 0.5$ can be violated while maintaining Stokesian settling behavior.

The limits of the quadratic Stokes law's applicability for speed are established with taking into account the density of the pulp. The inverse power dependence of the reduction of the numerical coefficient in the Stokes law on the density is determined. The power-law relationship between free and constrained settling velocities as pulp density increases is determined.

The obtained results expand scientific understanding of constrained settling processes. They are necessary for the development of new methods and devices for gravity beneficiation. Specifically, they are used to calculate the design parameters and operational modes of hydraulic separators for removing coal as a harmful impurity from fly ash allowing the ash to be used as a building material.

REFERENCES

1. Babii, K., Ikol, O., and Malieiev, Y. (2019), "Substantiating a technique to process hard rock involving extraction of valuable components and use of wastes to form mesorelief", *E3S Web of Conferences*, vol. 109, 00003 <https://doi.org/10.1051/e3sconf/201910900003>
2. Koshlak, G.V. and Pavlenko, A.M. (2021), "Prospects for the production of TES ash for the preparation of household materials", *Ecological safety and balanced resource consumption*, no 1 (23), pp. 92–101.
3. Khudyakova, L.I., Zalutskiy, A.V. and Paleev, P.L. (2019), "Use of ash and slag waste from thermal power plants", *J. XXI century. Technosphere safety*, no 4(3), pp. 375–391, <https://doi.org/10.21285/2500-1582-2019-3-375-391>
4. State building committee (1991), *Interstate standard GOST 25818-91. Thermal plant fly-ashes for concretes. Specification*, Moscow, USSA, 12 p., available at: <https://www.gost.rf.com/normadata/1/4294853/4294853144.pdf> (Accessed 12 August 2024).
5. Shevchenko, G.O., Usov, O.A., Cholyshkina, V.V. and Kurilov, V.S. (2024), "Efficiency of hydraulic separation of TTPs fly-ash", *Proceedings of the XXI International Scientific and Technical Conference "Poturaevskiy reading"*, 23 February, Dnipro: NTU "Dnipro Polytechnic", p. 36, available at: <https://gmi.nmu.org.ua/ua/nauka/vibro/%D0%A2%D0%B5%D0%B7%D0%B8%20%D0%9F%D0%A7%202024%201.pdf> (Accessed 17 June 2024).
6. Bogdanov, O.S. and Olevsky, V.A. (1972), *Spravochnik po Obogascheniju rud. Tom.1* [Handbook for ore dressing], vol.1, Nedra, Moscow, USSA, pp. 229–233.
7. Nadutyi, V.P., Kurilov, V.S., Cholyshkina, O.G. and Hankevych, V.F. (2021), "Analytical studies on constrained particle settling velocity in a water suspension of fly ash from thermal power plants", *Naukovyi Visnyk Natsionalnoho Hirnychoho Uni-versytetu*, no. 1(181), pp.32-39, <https://doi.org/10.33271/nvngu/2021-1/032>
8. Bulat, A.F., Shevchenko, G.O., Cholyshkina, V.V. and Kurilov, V.S. (2023), "Experimental determination of the solid phase density of thermal power plant ash", *Geo-Technical Mechanics*, no.167, pp. 110-118, <https://doi.org/10.15407/geotm2023.167.110>
9. Shevchenko, G.O., Cholyshkina, O.G., Sukharyev, V.V. and Kurilov, V.S. (2023), "Mathematical model for determining the hydraulic characteristics of finely dispersed water mineral suspensions", *Geo-Technical Mechanics*, no.163, pp. 155-164, <https://doi.org/10.15407/geotm2022.163.155>
10. Shevchenko, G.O., Cholyshkina, V.V., Sukharyev, V.V., Kurilov, V.S. and Lebed, H. B. (2023), "The rate of con-strained particles deposition in a wide range of suspension density in the laminar -turbulent operating modes", *Geo-Technical Mechanics*, no.164, pp. 103-116, <https://doi.org/10.15407/geotm2023.164.103>

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ЗАКОНОМІРНОСТІ СТИСНЕНОГО ОСАДЖЕННЯ ЧАСТИНОК У ВОДНИХ МІНЕРАЛЬНИХ СУСПЕНЗІЯХ РІЗНОЇ ГУСТИНИ

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Анотація. Швидкість осадження частинок в мінеральних суспензіях є важливим показником для розрахунку конструкції різноманітних гідравлічних апаратів і пристроїв для збагачення мінеральних пульп. При дослідженнях гравітаційного розділення різнорідних частинок шляхом осадження найменш дослідженими є питання визначення швидкості масового осадження, впливу густини суспензії на процес, застосовності класичних законів гідродинаміки. Нерідко для обчислення процесів гідравлічної сепарації застосовують умови вільного осадження, але це вносить значну похибку у величину швидкості бо на практиці процес протікає в стиснених умовах. Метою роботи був аналіз закономірностей стисненого осадження на прикладі осадження частинок вугілля в суспензіях золи виносу теплових електростанцій. В статті використана авторська методика розрахунку характеристик суспензій і швидкості стисненого осадження залежно від густини. Приведені експериментальні дані швидкості масового осадження натуральної золи виносу, котрі вказали на порядок швидкостей і аргументували розрахунок швидкості. Зважаючи,

що крупність золи мала, основна увага приділялась осадженню дрібного вугілля в золі. Аналізу підлягала база даних в якій при варіюванні густини суспензії золи $1,05 \div 1,3 \text{ г/см}^3$ і крупності осаджуваного вугілля $0,01 \div 4 \text{ мм}$ визначені характеристики суспензій і швидкості його стисненого осадження вугілля. Виконано аналіз бази даних за числом Рейнольдса і застосовністю законів Стокса і Лященко. Встановлено, що чим більш рідка суспензія, тим менша крупність частинок, рух яких підпорядковується закону Стокса і тим менший діапазон крупності закон Стокса охоплює і навпаки. Для дрібних класів вугілля $0,001\text{--}0,1 \text{ мм}$ чисельний коефіцієнт в законі Стокса залежно від густини пульпи зменшується по зворотному степеневому закону. Відношення швидкостей вільного і стисненого осадження залежно від густини зменшується за степеневим законом і аналогічно закону Лященко для порозності. Проведені дослідження розширюють наукові уявлення щодо протікання процесів стисненого осадження, облегшують інженерні розрахунки при проектуванні гідравлічних апаратів і оптимізації режимів їх роботи.

Ключові слова: суспензія, зола виносу, щільність, швидкість, стиснене осадження.